Discrete

Uniform
Values: $m \le k \le n$
Expectation: ______
Variance: $\frac{(n-m+1)^2-1}{12}$
P(X=k) = $\frac{1}{(m-n+1)}$

Discrete/Continuous Analogs

Uniform(a, b)
Values: $x \in (a, b)$
Expectation: ______
Variance: $\frac{(b-a)^2}{12}$
pdf: ______

Continuous

$U_1, \ldots U_n$ *iid* $Unif(0,1)$
$U_{(k)}$: ________
Fill in the distribution (don't forget the parameters)

____
Values: $0 \le x \le 1$
Expectation: $\frac{r}{r+s}$
Variance: $\frac{rs}{(r+s)^2(r+s+1)}$
pdf: $\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1}(1-x)^{s-1}$

____
Values: 0, 1
Expectation: p
Variance: pq
P(X = 1) = p
P(X=0) = q

$\sum_{i=1}^{n} X_i$ *iid*

$n = 2$

Binomial(n, p)
Values: $0 \le k \le n$
Expectation: np
Variance: npq
P(X = k) = $\binom{n}{k} p^k q^{n-k}$

____

____
Values: ________
Expectation: ____
P(X = k) = $\binom{n}{k} \frac{C(r,s)}{C(r+k, r+s-k)}$
$C(r,s) = \frac{\Gamma(r,s)}{\Gamma(r)\Gamma(s)}$

Number of heads in n tosses of a $p \sim beta(r,s)$ *coin*
Conditional distribution of X given p: $Binom(n,p)$

What's the difference Between Binomial and Hypergeometric? ________________

$k = 2$

Hypergeometric (N, G, n)
Values: $0 \le g \le n$
Expectation: ______
Variance: $n \frac{G}{N} \cdot \frac{N-G}{N} \cdot \frac{N-n}{N-1}$
P(X = g) = ________

What is the relationship between the Poisson and Binomial? ________ ________________

$Y = \rho X + \sqrt{1-\rho^2} Z$
$-1 < \rho < 1$
X, Z *iid* $N(0,1)$

$X, Y \sim$ **Standard Bivariate Normal** w correlation ρ

$\sigma Z + \mu$

$\sum X_i$

Normal(μ, σ^2)
Values: $-\infty \le x \le \infty$
Expectation: ____
Variance: ____
pdf: $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

Normal(0, 1)
Values: $-\infty \le x \le \infty$
Expectation: 0
Variance: 1
pdf: $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

Transform X to standard normal

$\sqrt{Z_1^2 + Z_2^2}$
Z_1, Z_2 *iid* $N(0,1)$

Rayleigh
Values: ______
Expectation: $\frac{\sqrt{\pi}}{2}$
Variance: $\frac{4-\pi}{2}$
pdf: $xe^{-\frac{1}{2}x^2}$

Z_1 / Z_2
Z_1, Z_2 *iid* $N(0,1)$

Cauchy
Values: ______
Expectation: undefined
Variance: undefined
pdf: $\frac{1}{\pi(1+x^2)}$

$\sum_{i=1}^{n} Z_i^2$ *iid* $N(0,1)$

Chi-Squared(n)
Values: ______
Expectation: n
Variance: $2n$
pdf: $Gamma(\frac{n}{2}, \frac{1}{2})$

Poisson(μ)
Values: ______
Expectation: __
Variance: ____
P(X = k) = ____

X_i: $Pois(\mu_i)$, *ind*
$\sum_{i=1}^{n} X_i : Pois(\mu_1 + \cdots \mu_n)$

n iid trials
k classes

____
$P(N_1 = n_1, \ldots N_k = n_k) = \frac{n!}{n_1! n_2! \ldots n_k!} p_1^{n_1} \ldots p_k^{n_k}$
N_i: $Binom(n, p_i)$

n: $Pois(\mu)$
N_i: $Poisson(\mu p_i)$
$N_1, \ldots N_k$ *ind*

Z: $N(0,1)$
______Z^2
Fill in the distribution

The Central Limit Theorem
Let $X_1, X_2, \ldots$ be iid., with mean μ and SD σ. Let $S_n = X_1 + X_2 + \ldots X_n$. Then, $E(S_n) = n\mu, SD(X_n) = \sqrt{n}\sigma$
For n → ∞, $S_n \approx N(n\mu, n\sigma^2)$
Why does this matter?
The normal distribution is a valid approximation for the sum of **any** iid random variables (for large enough n). In this way, the normal distribution is a central structure that connects all distributions.

Negative Binomial(r, p)
Values: $k \ge 0$
Expectation: $\frac{rq}{p}$
Variance: $\frac{rq}{p^2}$
P(X = k) = $\binom{k+r-1}{r-1} p^{r-1} q^k p$

$\sum_{i=1}^{r} X_i$ *iid* $Geom(p)$

Geometric(p)
Values: $k \ge 0$
Expectation: $\frac{q}{p}$
Variance: $\frac{q}{p^2}$
P(X = k) = $q^k p$
P(X > k) = q^{k+1}

Succeed on trial k

k failures before success

Geometric(p)
Values: $k \ge 1$
Expectation: ____
Variance: $\frac{q}{p^2}$
P(X = k) = $q^{k-1}p$
P(X > k) = q^k

Discrete/Continuous Analogs

____
Values: ________
Expectation: $\frac{r}{\lambda}$
Variance: $\frac{r}{\lambda^2}$
pdf: $\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$

$r, s = \frac{1}{2}$
$n = 1$

Exponential(λ)
Values: $k \ge 0$
Expectation: $\frac{1}{\lambda}$
Variance: $\frac{1}{\lambda^2}$
pdf: ________
Survival fn: ________

Fill in the distributions
$Exp(\lambda)$ = ________
$\sum_{i=1}^{r} X_i$ *iid*: ______

Recall:
$\Gamma(r) = (r-1)!, r \in \mathbb{Z}$
$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

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