

# Prob 140 Fall 2018 Final Exam Reference Sheet

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name and parameters	values	mass function or density	cdf $F$ or survival function	expectation	variance	mgf $M(t)$
Uniform	$m \leq k \leq n$	$1/(n-m+1)$		$(m+n)/2$	$((n-m+1)^2-1)/12$	
Bernoulli ( $p$ )	$0, 1$	$p_1 = p, p_0 = q$		$p$	$pq$	$q + pe^t$
Binomial ( $n, p$ )	$0 \leq k \leq n$	$\binom{n}{k} p^k q^{n-k}$		$np$	$npq$	$(q + pe^t)^n$
Poisson ( $\mu$ )	$k \geq 0$	$e^{-\mu} \mu^k / k!$		$\mu$	$\mu$	$\exp(\mu(e^t - 1))$
Geometric ( $p$ )	$k \geq 1$	$q^{k-1} p$	$P(X > k) = q^k$	$1/p$	$q/p^2$	
"Negative binomial" ( $r, p$ )	$k \geq r$	$\binom{k-1}{r-1} p^{r-1} q^{k-r} p$		$r/p$	$rq/p^2$	
Geometric ( $p$ )	$k \geq 0$	$q^k p$	$P(X > k) = q^{k+1}$	$q/p$	$q/p^2$	
Negative binomial ( $r, p$ )	$k \geq 0$	$\binom{k+r-1}{r-1} p^{r-1} q^k p$		$rq/p$	$rq/p^2$	
Hypergeometric ( $N, G, n$ )	$0 \leq g \leq n$	$\binom{g}{g} \binom{b}{b} / \binom{N}{n}$		$n \frac{G}{N}$	$n \frac{G}{N} \cdot \frac{B}{N} \cdot \frac{N-n}{N-1}$	
Uniform	$x \in (a, b)$	$1/(b-a)$	$F(x) = (x-a)/(b-a)$	$(a+b)/2$	$(b-a)^2/12$	
Beta ( $r, s$ )	$x \in (0, 1)$	$\frac{\Gamma(r+s)}{\Gamma(r)\Gamma(s)} x^{r-1} (1-x)^{s-1}$	$F(x)$ by uniform order statistics if $r, s$ integers	$r/(r+s)$	$rs/((r+s)^2(r+s+1))$	
Exponential ( $\lambda$ ) = Gamma ( $1, \lambda$ )	$x \geq 0$	$\lambda e^{-\lambda x}$	$F(x) = 1 - e^{-\lambda x}$	$1/\lambda$	$1/\lambda^2$	
Gamma ( $r, \lambda$ )	$x \geq 0$	$\frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}$		$r/\lambda$	$r/\lambda^2$	$(\lambda/(\lambda-t))^r, t < \lambda$
Chi-square ( $n$ )	$x \geq 0$	same as gamma ( $n/2, 1/2$ )		$n$	$2n$	
Normal ( $0, 1$ )	$x \in \mathbb{R}$	$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$		$0$	$1$	$\exp(t^2/2)$
Normal ( $\mu, \sigma^2$ )	$x \in \mathbb{R}$	$\frac{1}{\sigma} \phi((x-\mu)/\sigma)$		$\mu$	$\sigma^2$	
Rayleigh	$x \geq 0$	$xe^{-\frac{1}{2}x^2}$	$F(x) = 1 - e^{-\frac{1}{2}x^2}$	$\sqrt{\pi}/2$	$(4-\pi)/2$	
Cauchy	$x \in \mathbb{R}$	$1/\pi(1+x^2)$	$F(x) = \frac{1}{2} + \frac{1}{\pi} \arctan(x)$			

- If  $X_1, X_2, \dots, X_n$  are i.i.d. with variance  $\sigma^2$ , then  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2$  is an unbiased estimator of  $\sigma^2$  but  $\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2$  is not.
- For  $r > 0$ , the integral  $\Gamma(r) = \int_0^\infty x^{r-1} e^{-x} dx$  satisfies  $\Gamma(r+1) = r\Gamma(r)$ . So  $\Gamma(r) = (r-1)!$  if  $r$  is an integer. Also,  $\Gamma(1/2) = \sqrt{\pi}$ .
- If  $Z_1$  and  $Z_2$  are i.i.d. standard normal then  $\sqrt{Z_1^2 + Z_2^2}$  is Rayleigh. • If  $Z$  is standard normal then  $E(|Z|) = \sqrt{2/\pi}$
- The  $k$ th order statistic  $U_{(k)}$  is  $k$ th smallest of  $U_1, U_2, \dots, U_n$  i.i.d. uniform  $(0, 1)$ , so  $U_{(1)}$  is min and  $U_{(n)}$  is max. Density of  $U_{(k)}$  is beta  $(k, n-k+1)$ .
- If  $S_n$  is the number of heads in  $n$  tosses of a coin whose probability of heads was chosen according to the beta  $(r, s)$  distribution, then the distribution of  $S_n$  is beta-binomial  $(r, s, n)$  with  $P(S_n = k) = \binom{n}{k} C(r, s) / C(r+k, s+n-k)$  where  $C(r, s) = \Gamma(r+s) / (\Gamma(r)\Gamma(s))$  is the constant in the beta  $(r, s)$  density.
- If  $\mathbf{X}$  has mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\Sigma$  then  $\mathbf{A}\mathbf{X} + \mathbf{b}$  has mean vector  $\mathbf{A}\boldsymbol{\mu} + \mathbf{b}$  and covariance matrix  $\mathbf{A}\Sigma\mathbf{A}^T$ .
- If  $\mathbf{X}$  has the multivariate normal distribution with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\Sigma$ , then  $\mathbf{X}$  has density  $f(\mathbf{x}) = \frac{1}{(\sqrt{2\pi})^n \sqrt{\det(\Sigma)}} \exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu}))$
- The least squares linear predictor of  $Y$  based on the  $p \times 1$  vector  $\mathbf{X}$  is  $\hat{Y} = \mathbf{b}^T (\mathbf{X} - \boldsymbol{\mu}_X) + \mu_Y$  where  $\mathbf{b} = \Sigma_X^{-1} \Sigma_{X,Y}$ . Here the  $i$ th element of the  $p \times 1$  vector  $\Sigma_{X,Y}$  is  $\text{Cov}(X_i, Y)$ . In the case  $p = 1$  this is the equation of the regression line, with slope  $\text{Cov}(X, Y) / \text{Var}(X) = rSD(Y) / SD(X)$  and intercept  $E(Y) - \text{slope}E(X)$ .
- If  $W = Y - \hat{Y}$  is the error in the least squares linear prediction, then  $E(W) = 0$  and  $\text{Var}(W) = \text{Var}(Y) - \Sigma_{Y,X} \Sigma_X^{-1} \Sigma_{X,Y}$ . In the case  $p = 1$ ,  $\text{Var}(W) = (1-r^2)\text{Var}(Y)$ .
- If  $Y$  and  $\mathbf{X}$  are multivariate normal then the formulas in the above two bullet points are the conditional expectation and conditional variance of  $Y$  given  $\mathbf{X}$ .
- If  $Y$  and  $X$  are standard bivariate normal with correlation  $r$ , then  $Y = rX + \sqrt{1-r^2}Z$  for some standard normal  $Z$  independent of  $X$ .
- Under the multiple regression model  $\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$ , the least squares estimate of  $\boldsymbol{\beta}$  is  $\hat{\boldsymbol{\beta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Y}$ .