Prob 140 Final Practice Problems

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This is just meant to be a set of practice problems for topics not covered extensively in the Pitman text. This is definitely not intended to be comprehensive.

Maximum Likelihood Estimation

1. Suppose I draw $X_1, X_2, \ldots, X_n$ from a Poisson Distribution

$$f_X(x) = \frac{e^{-\theta}\theta^x}{x!};\ x \in \mathbb{N}_0, \theta > 0$$

Find $\hat{\theta}_{MLE}$. (Should be review)

2. Suppose I draw $X_1, X_2, \ldots, X_n$ from a Laplace Distribution

$$f_X(x) = \frac{1}{2}e^{-|x-\theta|};\ x, \theta \in \mathbb{R}$$

Find $\hat{\theta}_{MLE}$.

3. Suppose I have the following discrete distribution:

x	0	1	2	3
$P(X = x)$	$\frac{2\theta}{3}$	$\frac{\theta}{3}$	$\frac{2(1-\theta)}{3}$	$\frac{1-\theta}{3}$

where $0 \leq \theta \leq 1$. I sample from the distribution 10 times and get two 0's, three 1's, three 2's and two 3's. Find $\hat{\theta}_{MLE}$

Moment Generating Functions

Recall that the moment generating function is defined as $M_X(t) = E(e^{tX})$. For discrete distributions, $M_X(t) = \sum_{i=1}^{\infty} e^{tx_i} p_i$ and for continuous distributions, $M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$

1. Find the MGF for a Poisson distribution and use it to prove that the sum of Poisson distributions is also Poisson.

2. Find the MGF of the Standard Normal Distribution. (Hint: expand $(x - t)^2$)

3. Find the MGF of the uniform (a, b) and use it to find the expectation and variance.

Convolution

If X and Y have density $f(x, y)$, then

$$f_{X+Y}(z) = \int_{-\infty}^{\infty} f(x, z - x)dx$$

Note that if X and Y are independent, then $f(x, z - x) = f_X(x)f_Y(z - x)$. Also note that if X and Y are non-negative, then the bounds become $(0, z)$ instead of $(-\infty, \infty)$

1. Let $X, Y \sim expon(\lambda)$. Find the distribution of $X + Y$. (Pitman 373)
2. Let $X \sim gamma(r, \lambda), Y \sim gamma(s, \lambda)$. Find the distribution of $X + Y$ (Pitman 376)

CLT

1. Suppose I have a coin that lands heads with probability p=0.75. I flip the coin 10000 times. What is the probability that I got a heads at least 8000 times?
2. We discovered in lab that the number of soldiers per cavalry corp killed by a horse kick each year is approximately modeled as a Poisson Distribution with $\mu = 0.7$. Suppose over a 200 year period, Prussia fielded an average of 12.5 cavalry corps. What's the probability that fewer than 2000 Prussian cavalrymen died of a horse kick over that period?